7. The Solow model (cont.)

**7.1** Suppose an economy which functions in accordance with the hypotheses of the Solow model, with the following Cobb-Douglas production function:

Y = 0,4.Kα.L1-α

We also know that the partial elasticity of GDP with respect to physical capital is 0.6, the depreciation rate is 4%, the labour force growth rate is 0.5% and the steady state level of physical capital per worker is 7.36 monetary units.

a) Find the steady state value of GDP assuming that the labour force consists of 10 million people.

**n=0.005**

**δ=0.04**

**α=0.6**

**k\*=7.36**

**L = 10m**

***Y* = 0,4.*K*0.6.*L*0.4**

**y = 0.4.k0.6**

**Y\*=?**

**y\* = 0.4.k\*0.6 = 0.4.(7.36)0.6 = 1.32~**

**Y\* = y\*.L = 109\*1.32~ = 13.2m~**

b) Find the level of the investment rate which is consistent with this steady state level of physical capital per worker.

**s=?**

**Condição de equilíbrio: sy = (n+δ).k**

**s\*(1.32) = 0.045\*7.36**

**s = 0.25~ = 25%~**

c) Provide an economic explanation for the mechanism which leads the stock of physical capital per worker to return to its steady state level if at any time it increases above 7.36. What does this imply in relation to the ability of capital accumulation to bring about sustained productivity increases in the Solow model?

**If k exceeds k\*, then (n+δ)\*k, i.e. the investment required just to maintain the capital stock per worker constant will exceed actual investment s.y. Consequently, net investment will be negative. The process will continue until k = k\*, at which point s.y = (n+δ).k.**



**7.2** Suppose an Economy A which functions in accordance with the hypotheses of the Solow model, with the following Cobb-Douglas production function:

Y = 0,3.K0,4.L0,6

The investment rate is 20%, the labour force growth rate is 1% and the depreciation rate is 4%. We also know the initial values of K and L, which are as follows: K0 = 11 million monetary units; and L0 = 10 million people.

a) What is the future trend of the capital-output ratio in this economy? And what is the future trend of labour productivity?

**K/Y = K/0.3\*K0.4\*L0.6 = (1/0.3)\*(K0.6/L0.6) = 1/0.3\*k0.6 (depende positivamente de k). Logo, o que queremos saber é qual a tendência de evolução de k, o que depende de k0 estar acima ou abaixo de k\*. O mesmo sucede com a produtividade do trabalho, y, que depende positivamente de k segundo a fórmula y = 0.3\*k0.4.**

**Temos então k0 = 11m/10m = 1.1**

**k\*=?**

**Condição de equilíbrio: s.y = (n+δ)k**

**0.2(0.3.k0.4) = (0.01+0.04).k**

**0.06\*k0.4 = 0.05k**

**0.06\*k0.4 - 0.05k = 0**

**k\*(0.06\*k-0.6 - 0.05) = 0**

**0.06\*k-0.6 - 0.05 = 0**

**k = (0.05/0.06)-1/0.6 =1.36~**

**k tem tendência a aumentar de k=1.1 para k\*=1.36. Logo, também K/Y e Y/L terão tendência a aumentar. Pode-se aliás calcular os seus níveis atuais e de steady state a partir das fórmulas deduzidas em cima.**

b) Assume that this economy takes 20 years to reach the steady state. Characterize this equilibrium state in terms of the level and growth rates of the following variables: labour force, stock of physical capital, GDP, investment.

**Daqui a vinte anos, no steady state:**

**L = 109\*(1.01)20 = 12.2m~**

**K = k\*L = 1.36\*12.2m = 16.6m~**

**y = 0.3\*k0.4 = 0.3\*1.360.4 = 0.34~ Logo, Y = y\*L = 0.34\*12.2m = 4.15m~**

**I = s.Y = 0.2\*4.15m = 0.83m~**

c) Consider that this economy is in the steady state and the government of this country decides to create a special income tax, which consists of a tax of 5% on GDP. What are the consequences upon the level and growth rate of GDP per worker?

**A taxa de investimento s passa agora a incidir sobre um rendimento disponível menor em 5%. É como se tivéssemos uma nova taxa de poupança, mais baixa:**

**s=0.2 , t = 0.05 🡪 s’ = 0.2(1-t) = 0.2(0.95) = 0.19**

**Novo estado estacionário:**

**0.19(0.3.k0.4) = (0.01+0.04).k**

**0.057\*k0.4 = 0.05k**

**0.057\*k0.4 - 0.05k = 0**

**k\*(0.057\*k-0.6 - 0.05) = 0**

**0.057\*k-0.6 - 0.05 = 0**

**k = (0.05/0.057)-1/0.6 =1.24~**

**y= 0.3\*k0.4 = 0.3\*1.240.4 = 0.33~ (era 0.34~ ).**

**A taxa de crescimento de y permanece inalterada e é nula, pois estamos no estado estacionário.**

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d) What are the effects, both in the short term and the long term, of a one-off increase in the labour force to 11 million people (for example due to the massive return of refugees in a given year)?

**In the short term, the one-off population increase will reduce the level of k (=K/L) and consequently the level of y (which depends on k). If the economy was previously in the steady state, this will bring it to the left of the steady state, initiating the usual adjustment process back to k\*. In the long term, the economy will be back in the steady state, which is itself unchanged given that neither the production function, the savings rate, the population growth rate and the depreciation rate are unchanged.**

**7.3** Consider an economy which behaves in accordance with the hypotheses of the Solow model and about which it is known that the investment rate equals 25%, the depreciation rate equals 4% and the labour force is constant. The production function of this economy is as follows:

Yt = Kt0.4.Lt0.6

a) Find the steady state level of physical capital per worker of this economy. In addition, find the level of labour productivity which corresponds to that steady state.

**y = k0.4**

**s=0.25**

**δ=0.04**

**n=0**

**k\*=?**

**Condição de equilíbrio no steady state:**

**s.y=(n+ δ)k**

**0.25\*k0.4 = 0.04k**

**0.25\*k0.4 - 0.04k = 0**

**k\*(0.25k-0.6 – 0.04) = 0**

**k-0.6 = 0.04/0.25**

**k = (0.04/0.25)-1/0.6 = 21.1~**

b) Suppose that, starting from the steady state, the government of this country is considering two alternative immigration policies. The first one consists of the immigration of a number of migrants corresponding to 10% of the population in one go, albeit without any effect upon the population growth rate thereafter (which continues to be zero). The second one consists of the immigration every year of a number of migrants such that the population growth rate undergoes a permanent increase from 0% to 2%. Explain the consequences of these two alternatives in terms of the Solow model and depict the two situations in a graph.

**Nos termos do modelo de Solow, a primeira alternativa altera o nível de L de uma só vez, aumentando-o e consequentemente reduzindo k = K/L para um nível inferior ao do estado estacionário, o que desencadeia o habitual porocesso de ajustamento de regresso a k\*. Nesse primeiro caso, o estado estacionário k\* permanece inalterado, pois não se alterarem os seus determinantes fundamentais. No segundo caso, estamos perante um aumento permanente da taxa de crescimento da população, n, o que altera o estado estacionário para um nível menor (graficamente, corresponde a uma rotação para cima da semireta do investimento necessário, conduzindo a uma interseção que ocorre a um nivel menor de k\*). Consequentemente, nesse segundo caso o que temos é um ajustamento do estado estacionário original k\* para um novo estado estacionário k\*’ inferior.**

**7.4** Consider an economy which behaves according to the Solow model and has the following aggregate production function:

Y = A.Kα.(h.L)(1-α)

The variables Y, K, h and L stand for the level of output, physical capital, human capital per worker and the labour force. The variable A, which represents the level of technology, is constant and equal to 1, while the elasticity of GDP with respect to physical capital is equal to 0.8.

Assuming that human capital per worker remains constant and equal to 1, that the depreciation rate is 3.5% per year, and that the labour force grows at 0.5% per year, compute the investment rate which is compatible with a steady state capital-output ratio equal to 4.

**Resposta:**

**Como habitualmente, partamos da função de produção para obtermos a função de produção por trabalhador com que trabalhamos habitualmente no modelo de Solow:**

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**Com *A*=1, *h*=1 e *α*=0,8, vem**

**, de onde retiramos**



**Queremos o valor de *s* compatível com K/Y=4**

***s*=? => K/Y=4**

**K/Y = 4 ⬄**

**⬄ K/(K0,8.L0,2) = 4 ⬄**

**⬄K0,2/L0,2=4 ⬄**

**⬄ *k*0,2= 4**

**⬄ *k* = 41/0,2 = 1024**

**Cálculo do valor de s compatível com *k* = 1024 no estado estacionário:**

***sy = (n+d)k***

***s*.10240,8 = (0,005+0,035).1024**

***s* = (0,005+0,035).1024**

***s* = 0,16 = 16%**

**A taxa de investimento compatível com uma situação de estacionariedade em que K/Y=4 (logo, em que *k* = 1024) é de *s* = 16%.**